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A new generalized KdV equation: Its lump-type, complexiton and soliton solutions

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A new generalized KdV equation, describing the motions of long waves in shallow water under the gravity field, is considered in this paper. By adopting a series of well-organized methods, the Bäcklund transformation, the bilinear form and diverse wave structures of the governing model are formally extracted. The exact solutions listed in this paper are categorized as lump-type, complexiton, and soliton solutions. To exhibit the physical

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mechanism of the obtained solutions, several graphical illustrations are given for particular choices of the involved parameters. As a direct consequence, diverse wave structures given in this paper enrich the studies on the KdV-type equations.

Keywords: New generalized KdV equation; well-organized methods; Bäcklund transformation; bilinear form; lump-type; complexiton; and soliton solutions.

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1. Introduction

Partial differential equations (PDEs) emerge in a wide variety of scientific fields and are capable tools for modeling many phenomena in optical fibers, ion-acoustic waves, and water waves. One of the most fundamental goals in much of today's research is searching for exact solutions to PDEs. Such a goal is crucial because exact solutions enable researchers to achieve valuable information and more insight into the phenomena under study. Nowadays, with the advancement of symbolic packages, several effective methods to find out exact solutions of PDEs such as the Kudryashov method,[1–4] the $\exp_a$ method,[5–8] and the modified Jacobi method[9–12] have been established. All of these show the importance of finding exact solutions to PDEs and their dynamical analysis.

Researchers are faced with a variety of exact solutions, such as lump, complexiton, and soliton solutions. Each of these types of exact solutions has its definition and those interested can refer to the papers[13–30] for more information. In the last few decades, such types of exact solutions have been the key subject of a lot of studies. For example, Sulaiman *et al.*[25] obtained lump solutions of a nonlinear PDE in $(3+1)$-dimensions using a test function. Zhou *et al.* in Ref. 26 applied an ansatz to derive lump solutions of a 2D Boussinesq-type equation. The authors of Ref. 27 found complexiton solutions of the KdV equation using the Hirota method. In a study conducted by Hosseini *et al.*,[28] the Hirota method was used to retrieve complexiton solutions of a Hirota equation. Wazwaz[29] obtained solitons of the KdV equations using the simplified Hirota method. Recently, the author of Ref. 30 adopted the simplified Hirota method to acquire solitons of the sinh-Gordon equations.

The main goal of this paper is to consider the following new generalized KdV equation describing the motions of long waves in shallow water under the gravity field

$$\frac{\partial u}{\partial t}+\frac{\partial^5 u}{\partial x^5}+15u\frac{\partial^3 u}{\partial x^3}+15\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+45u^2\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0 \tag{1}$$

and derive its lump-type, complexiton, and soliton solutions. For those interested, the $(2+1)$-dimensional version of Eq. (1) was formally proposed by Sun *et al.* in Ref. 31. It is noteworthy that Lü *et al.*[32] obtained lump and interaction solutions of $(2+1)$-dimensional generalized (2DG) KdV equation using the Hirota method. In another research, Liu[33] used an ansatz to derive interaction solutions of the 2DG

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KdV equation. Very recently, Yusuf and Sulaiman in Ref. 34 retrieved lump-periodic and other exact solutions of the 2DG KdV equation by adopting various methods.

This paper is organized as follows. In Sec. 2, by considering the model, its Bäcklund transformation and bilinear form are derived. In Sec. 3, by applying several well-established methods, diverse wave structures of the model, categorized as lump-type, complexiton, and soliton solutions, are formally extracted. Besides, to exhibit the physical mechanism of the obtained solutions, diverse graphical illustrations are taken in Sec. 3 for particular choices of the involved parameters. In the last section, a full discussion of the results is provided.

2. New Generalized KdV Equation: Its Bäcklund Transformation and Bilinear Form

To arrive at the Bäcklund transformation of the model, the authors utilize the truncated Painlevé expansion (TPE).[35–37] Owing to the TPE, the following solution to Eq. (1) is considered:

$$u = \frac{u_0}{\phi^2} + \frac{u_1}{\phi} + u_2. \tag{2}$$

In the above equation, u_2 satisfies the model, and u_0 and u_1 are established later.

Setting Eq. (2) in Eq. (1) and considering the coefficient of ϕ^{-7} to zero gives

$$-720u_0\left(\frac{\partial\phi}{\partial x}\right)^5 - 540u_0^2\left(\frac{\partial\phi}{\partial x}\right)^3 - 90u_0^3\frac{\partial\phi}{\partial x} = 0,$$

which its solution leads to

$$u_0 = -2\left(\frac{\partial\phi}{\partial x}\right)^2.$$

After taking $u_0 = -2(\frac{\partial\phi}{\partial x})^2$ and $u_2 = 0$, and considering the coefficient of ϕ^{-6} to zero, we derive

$$-360\left(\frac{\partial\phi}{\partial x}\right)^5\frac{\partial^2\phi}{\partial x^2} + 180u_1\left(\frac{\partial\phi}{\partial x}\right)^5 = 0.$$

From the above equation, u_1 is gained as

$$u_1 = 2\frac{\partial^2\phi}{\partial x^2}.$$

Now, $u_0 = -2(\frac{\partial\phi}{\partial x})^2$, $u_1 = 2\frac{\partial^2\phi}{\partial x^2}$, and $u_2 = 0$ result in

$$u = -2\frac{\left(\frac{\partial\phi}{\partial x}\right)^2}{\phi^2} + 2\frac{\frac{\partial^2\phi}{\partial x^2}}{\phi} = 2\frac{\partial^2\ln(\phi)}{\partial x^2}.$$

It is worth mentioning that the new generalized KdV equation can be expressed in the operator form as

$$(D_tD_x + D_xD_y + D_xD_z + D_x^6)\phi\cdot\phi = 0.$$

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The corresponding bilinear form of the above operator form is in the form

$$\begin{aligned}&2(\phi\phi_{xt}-\phi_x\phi_t)+2(\phi\phi_{xy}-\phi_x\phi_y)+2(\phi\phi_{xz}-\phi_x\phi_z)+2\phi\phi_{xxxxxx}\\&\quad-12\phi_x\phi_{xxxxx}+30\phi_{xx}\phi_{xxxx}-20\phi_{xxx}^2=0.\end{aligned}\tag{3}$$

3. New Generalized KdV Equation: Its Lump-Type, Complexiton, and Soliton Solutions

In this section, through considering several well-designed methods, diverse wave structures of the model, categorized as lump-type, complexiton, and soliton solutions, are formally extracted. Besides, to exhibit the physical mechanism of the obtained solutions, a number of graphical illustrations are taken for particular choices of the involved parameters.

3.1. *Lump-type solution of the model*

To retrieve the lump-type solution of the model, an ansatz is adopted as follows:

$$\phi=(a_1x+a_2y+a_3z+a_4t+a_5)^2+(a_6x+a_7y+a_8z+a_9t+a_{10})^2+a_{11},\tag{4}$$

where a_i, $i=1,2,\ldots,11$ are derived later. By inserting Eq. (4) into Eq. (3) and arranging the terms, we will achieve a nonlinear algebraic system whose solution leads to

$$a_2=-a_3-a_4,\quad a_7=-a_8-a_9.$$

Now, the following lump-type solution to the model is gained:

$$u=2\frac{\partial^2\ln(\phi)}{\partial x^2},$$

where

$$\begin{aligned}\phi&=(a_1x-(a_3+a_4)y+a_3z+a_4t+a_5)^2\\&\quad+(a_6x-(a_8+a_9)y+a_8z+a_9t+a_{10})^2+a_{11}.\end{aligned}$$

The above lump-type solution for $a_1=1$, $a_3=1$, $a_4=1$, $a_5=1$, $a_6=1$, $a_8=-1$, $a_9=2$, $a_{10}=1$, $a_{11}=1$, $y=1$, and $t=0$ can be written as

$$u=\frac{8}{(x+z-1)^2+(x-z)^2+1}-\frac{2(4x-2)^2}{((x+z-1)^2+(x-z)^2+1)^2}.$$

The physical mechanism of the above rational function which is a lump-type wave has been represented in Fig. 1.

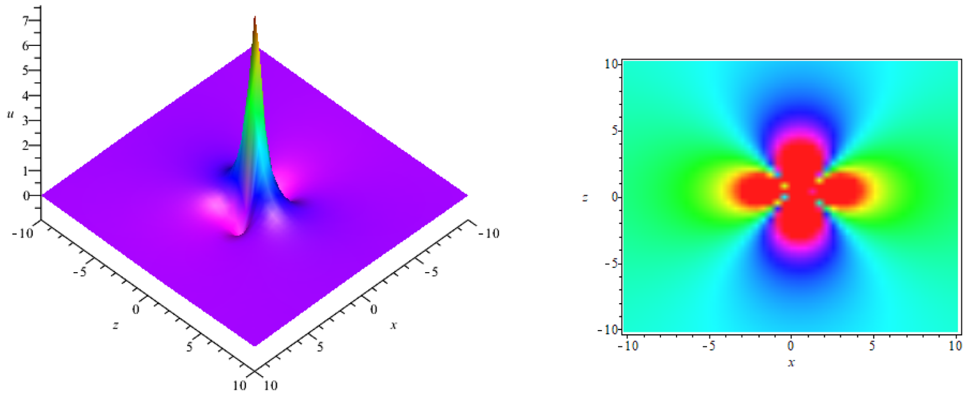

Fig. 1. (Color online) Lump-type solution for $a_1 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $a_6 = 1$, $a_8 = -1$, $a_9 = 2$, $a_{10} = 1$, $a_{11} = 1$, $y = 1$, and $t = 0$.

3.2. *Complexiton solution of the model*

To find the complexiton solution of the model, we first introduce

$$\begin{aligned} a &= a_1 + ia_2, \\ b &= b_1 + ib_2, \\ c &= c_1 + ic_2, \\ w &= w_1 + iw_2, \\ p &= xt + xy + xz + x^6. \end{aligned}$$

By considering the above assumptions and the following equations:

$$\begin{aligned} p(a, b, c, w) &= 0, \\ p(\bar{a}, \bar{b}, \bar{c}, \bar{w}) &= 0, \end{aligned}$$

a system is generated as follows:

$$\begin{aligned} 6a_1^5a_2 - 20a_1^3a_2^3 + 6a_1a_2^5 + a_1b_2 + a_1c_2 + a_1w_2 + a_2b_1 + a_2c_1 + a_2w_1 &= 0, \\ a_1^6 - 15a_1^4a_2^2 + 15a_1^2a_2^4 - a_2^6 + a_1b_1 + a_1c_1 + a_1w_1 - a_2b_2 - a_2c_2 - a_2w_2 &= 0, \\ -6a_1^5a_2 + 20a_1^3a_2^3 - 6a_1a_2^5 - a_1b_2 - a_1c_2 - a_1w_2 - a_2b_1 - a_2c_1 - a_2w_1 &= 0, \\ a_1^6 - 15a_1^4a_2^2 + 15a_1^2a_2^4 - a_2^6 + a_1b_1 + a_1c_1 + a_1w_1 - a_2b_2 - a_2c_2 - a_2w_2 &= 0. \end{aligned}$$

The above system can be solved to derive w_1 and w_2 as

$$\begin{aligned} w_1 &= -a_1^5 + 10a_1^3a_2^2 - 5a_1a_2^4 - b_1 - c_1, \\ w_2 &= -5a_1^4a_2 + 10a_1^2a_2^3 - a_2^5 - b_2 - c_2. \end{aligned}$$

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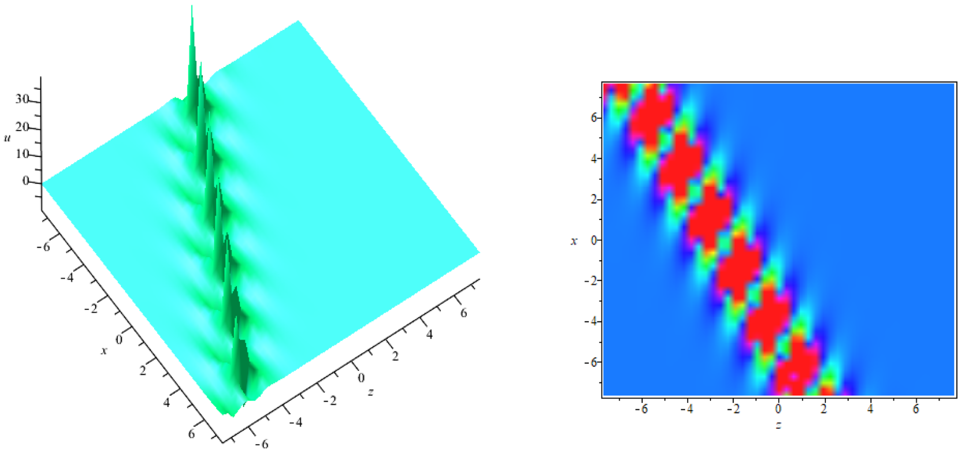

Fig. 2. (Color online) Complexiton solution for $a_1 = -1$, $b_1 = -4.95$, $c_1 = -2$, $a_2 = 1$, $b_2 = -0.75$, $c_2 = -3$, $y = 1$, and $t = 0$.

The unknown a_{12} is obtained by adopting the following formula:

$$a_{12} = -\frac{p(2ia_2, 2ib_2, 2ic_2, 2iw_2)}{p(2a_1, 2b_1, 2c_1, 2w_1)},$$

as

$$a_{12} = -\frac{-64a_2^6 - 4(-5a_1^4 a_2 + 10a_1^2 a_2^3 - a_2^5 - b_2 - c_2)a_2 - 4a_2 b_2 - 4a_2 c_2}{64a_1^6 + 2(-2a_1^5 + 20a_1^3 a_2^2 - 10a_1 a_2^4 - 2b_1 - 2c_1)a_1 + 4a_1 b_1 + 4a_1 c_1}.$$

Now, the complexiton solution to the model is acquired as

$$u = 2\frac{\partial^2 \ln(\phi)}{\partial x^2}, \quad \phi = 1 + 2e^{\vartheta_1}\cos(\vartheta_2) + a_{12}e^{2\vartheta_1},$$

where

$$\begin{aligned}
\vartheta_1 &= a_1 x + b_1 y + c_1 z + w_1 t, \\
\vartheta_2 &= a_2 x + b_2 y + c_2 z + w_2 t, \\
w_1 &= -a_1^5 + 10a_1^3 a_2^2 - 5a_1 a_2^4 - b_1 - c_1, \\
w_2 &= -5a_1^4 a_2 + 10a_1^2 a_2^3 - a_2^5 - b_2 - c_2, \\
a_{12} &= -\frac{-64a_2^6 - 4(-5a_1^4 a_2 + 10a_1^2 a_2^3 - a_2^5 - b_2 - c_2)a_2 - 4a_2 b_2 - 4a_2 c_2}{64a_1^6 + 2(-2a_1^5 + 20a_1^3 a_2^2 - 10a_1 a_2^4 - 2b_1 - 2c_1)a_1 + 4a_1 b_1 + 4a_1 c_1}.
\end{aligned}$$

The physical mechanism of the above complexiton which is a combination of exponential and trigonometric waves has been shown in Fig. 2 for $a_1 = -1$, $b_1 = -4.95$, $c_1 = -2$, $a_2 = 1$, $b_2 = -0.75$, $c_2 = -3$, $y = 1$, and $t = 0$.

3.3. *Soliton solutions of the model*

To construct soliton solutions of the model, the nonlinear terms of Eq. (1) are first removed. This yields

$$\frac{\partial u}{\partial t} + \frac{\partial^5 u}{\partial x^5} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0. \tag{5}$$

By assuming the solution of Eq. (5) as

$$u = e^{\vartheta_i}, \quad \vartheta_i = a_i x + b_i y + c_i z - w_i t,$$

we will obtain

$$(a_i^5 + b_i + c_i - w_i)e^{a_i x + b_i y + c_i z - w_i t} = 0.$$

From the above equation, the dispersion relation w_i is found as

$$w_i = a_i^5 + b_i + c_i.$$

Consequently, we can introduce the following phase variables:

$$\vartheta_i = a_i x + b_i y + c_i z - (a_i^5 + b_i + c_i)t, \quad i = 1, 2.$$

Now, the following ansatz:

$$u = R\frac{\partial^2 \ln(\phi)}{\partial x^2}, \quad \phi = 1 + e^{a_1 x + b_1 y + c_1 z - (a_1^5 + b_1 + c_1)t},$$

is inserted into the new generalized KdV equation, yielding $R = 2$.

Thus, the single soliton to the model is constructed as

$$u = 2\frac{\partial^2 \ln(\phi)}{\partial x^2},$$

where

$$\phi = 1 + e^{a_1 x + b_1 y + c_1 z - (a_1^5 + b_1 + c_1)t}.$$

To arrive at the double soliton of the model, the following ansatz:

$$u = 2\frac{\partial^2 \ln(\phi)}{\partial x^2}, \quad \phi = 1 + e^{\vartheta_1} + e^{\vartheta_2} + a_{12}e^{\vartheta_1 + \vartheta_2},$$

is substituted into the new generalized KdV equation. This leads to the phase shift as follows:

$$a_{12} = \frac{a_1^4 - 3a_1^3 a_2 + 4a_1^2 a_2^2 - 3a_1 a_2^3 + a_2^4}{a_1^4 + 3a_1^3 a_2 + 4a_1^2 a_2^2 + 3a_1 a_2^3 + a_2^4}.$$

So, the double soliton to the model is established as

$$u = 2\frac{\partial^2 \ln(\phi)}{\partial x^2}, \quad \phi = 1 + e^{\vartheta_1} + e^{\vartheta_2} + a_{12}e^{\vartheta_1 + \vartheta_2},$$

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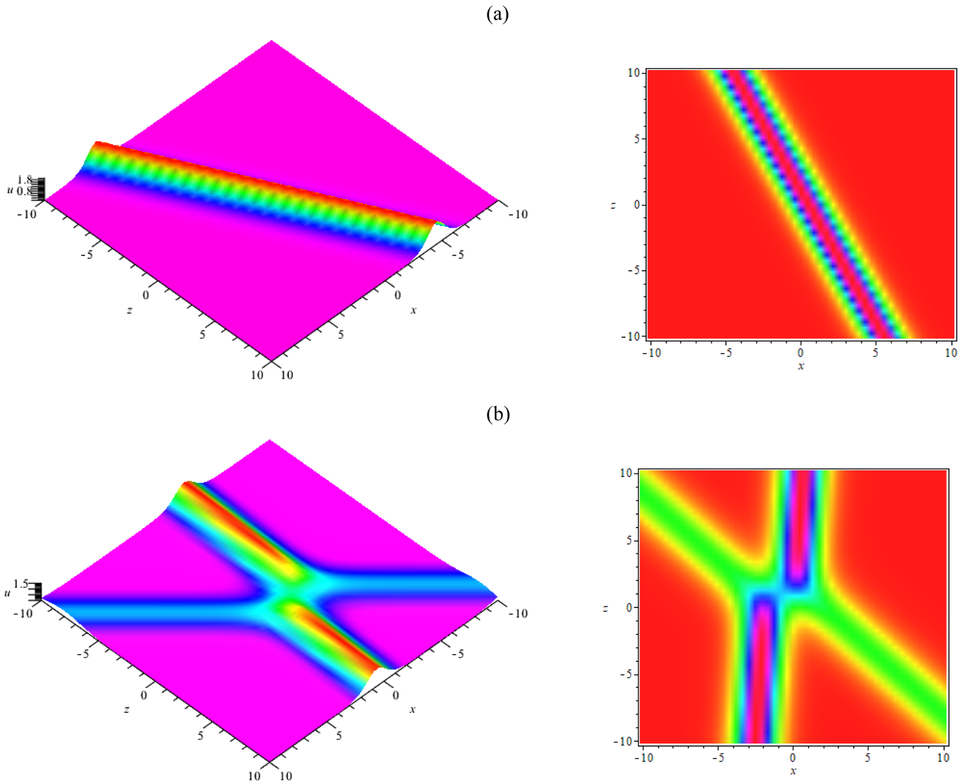

Fig. 3. (Color online) (a) Single soliton for $a_1 = -2$, $b_1 = 1$, $c_1 = -1$, $y = 1$, and $t = 0$; (b) Double soliton for $a_1 = -1.75$, $b_1 = 0.1$, $c_1 = 0.1$, $a_2 = -1$, $b_2 = 2$, $c_2 = -1$, $y = 1$, and $t = 0$.

where

$$\phi = 1 + e^{\vartheta_1} + e^{\vartheta_2} + a_{12}e^{\vartheta_1+\vartheta_2},$$

$$\vartheta_i = a_i x + b_i y + c_i z - (a_i^5 + b_i + c_i)t, \quad i = 1, 2,$$

$$a_{12} = \frac{a_1^4 - 3a_1^3a_2 + 4a_1^2a_2^2 - 3a_1a_2^3 + a_2^4}{a_1^4 + 3a_1^3a_2 + 4a_1^2a_2^2 + 3a_1a_2^3 + a_2^4}.$$

Figure 3 represents the physical mechanism of the above single and double solitons for (a) $a_1 = -2$, $b_1 = 1$, $c_1 = -1$, $y = 1$, and $t = 0$; (b) $a_1 = -1.75$, $b_1 = 0.1$, $c_1 = 0.1$, $a_2 = -1$, $b_2 = 2$, $c_2 = -1$, $y = 1$, and $t = 0$. More precisely, Fig. 3(a) shows a bright wave while Fig. 3(b) demonstrates the interaction of two bright waves.

3.4. *Other solitons of the model*

To derive other single solitons of the model, we first adopt two ansatzes as follows:

$$\text{(i)} \quad u = A + B\tanh^2(ax + by + cz - wt),$$

$$\text{(ii)} \quad u = A + B\,\mathrm{sech}^2(ax + by + cz - wt).$$

By substitution of the first ansatz into Eq. (1), the following system is acquired:

$$360a^5 + 270Ba^3 + 45B^2a = 0,$$
$$-240a^5 - 180Aa^3 - 300Ba^3 - 90ABa - 90B^2a = 0,$$
$$16a^5 + 60Aa^3 + 60Ba^3 + 45A^2a + 90ABa + 45B^2a + b + c - w = 0,$$

with the following results:

Result 1:

$$B = -2a^2, \quad w = 76a^5 - 120Aa^3 + 45A^2a + b + c.$$

So, the following single soliton to the model is achieved:

$$u_1 = A - 2a^2 \tanh^2(ax + by + cz - (76a^5 - 120Aa^3 + 45A^2a + b + c)t).$$

Result 2:

$$A = \frac{8}{3}a^2, \quad B = -4a^2, \quad w = 16a^5 + b + c.$$

Therefore, the following single soliton to the model is achieved:

$$u_2 = \frac{8}{3}a^2 - 4a^2 \tanh^2(ax + by + cz - (16a^5 + b + c)t).$$

In a similar way, by substitution of the second ansatz into Eq. (1), the following system is derived:

$$360a^5 - 270Ba^3 + 45B^2a = 0,$$
$$-240a^5 - 180Aa^3 + 120Ba^3 + 90ABa = 0,$$
$$16a^5 + 60Aa^3 + 45A^2a + b + c - w = 0,$$

whose solution leads to the following results:

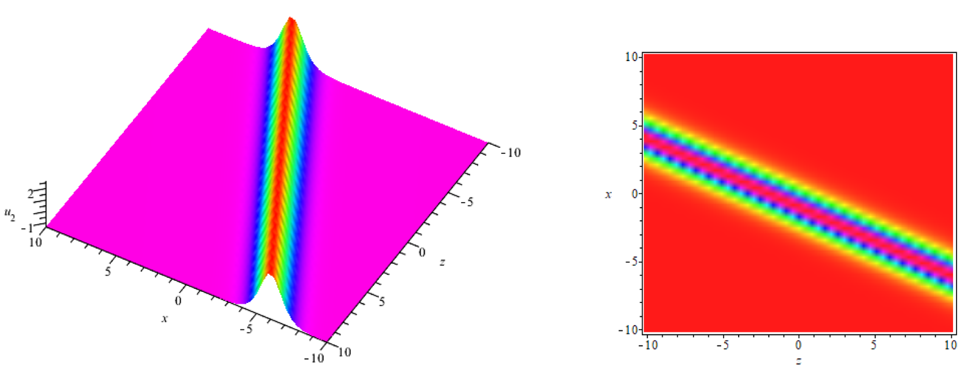

Fig. 4. (Color online) The second soliton for $a = 1$, $b = 1$, $c = 0.5$, $y = 1$, and $t = 0$.

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Result 1:

$$B = 2a^2, \quad w = 16a^5 + 60Aa^3 + 45A^2a + b + c.$$

Thus, the following single soliton to the model is gained:

$$u_3 = A + 2a^2 \mathrm{sech}^2(ax + by + cz - (16a^5 + 60Aa^3 + 45A^2a + b + c)t).$$

Result 2:

$$A = -\frac{4}{3}a^2, \quad B = 4a^2, \quad w = 16a^5 + b + c.$$

Consequently, the following single soliton to the model is obtained:

$$u_4 = -\frac{4}{3}a^2 + 4a^2 \mathrm{sech}^2(ax + by + cz - (16a^5 + b + c)t).$$

The physical mechanism of the second soliton, revealing a bright wave, has been shown in Fig. 4 for $a = 1$, $b = 1$, $c = 0.5$, $y = 1$, and $t = 0$.

4. Conclusion

The main goal of this paper was to study a new generalized KdV equation that simulates the motions of long waves in shallow water under the gravity field. More precisely, by applying the TPE, the Bäcklund transformation of the model was first extracted. Such a logarithmic transformation, $u = 2(\ln(\phi))_{xx}$, was then employed to construct the bilinear form of the new generalized KdV equation. Furthermore, several well-designed methods were formally adopted to acquire diverse wave structures of the governing model that are categorized as lump-type, complexiton, and soliton solutions. In the end, some graphical illustrations were considered to exhibit the physical mechanism of the obtained solutions for particular choices of the involved parameters. In future work, the authors will try to apply other methods[38–52] for constructing other diverse wave structures of the new generalized KdV equation.

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