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NONLINEAR PHYSICS AND MECHANICS

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A Study of Different Wave Structures of the $(2+1)$-dimensional Chiral Schrödinger Equation

K. Hosseini, M. Mirzazadeh, K. Dehingia, A. Das, S. Salahshour

In the present paper, the authors are interested in studying a famous nonlinear PDE referred to as the $(2+1)$-dimensional chiral Schrödinger (2D-CS) equation with applications in mathematical physics. In this respect, the real and imaginary portions of the 2D-CS equation are firstly derived through a traveling wave transformation. Different wave structures of the 2D-CS equation, classified as bright and dark solitons, are then retrieved using the modified Kudryashov (MK) method and the symbolic computation package. In the end, the dynamics of soliton solutions is investigated formally by representing a series of 3D-plots.

Keywords: $(2+1)$-dimensional chiral Schrödinger equation, traveling wave transformation, modified Kudryashov method, different wave structures

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Kamyar Hosseini
kamyar_hosseini@yahoo.com
Department of Mathematics, Rasht Branch, Islamic Azad University
P.O. Box 41335-3516 Rasht, Iran
Department of Mathematics, Near East University TRNC
Mersin 10, Nicosia 99138, Turkey

Mohammad Mirzazadeh
mirzazadehs2@guilan.ac.ir
Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan
P.C. 44891-63157 Rudsar-Vajargah, Iran

Kaushik Dehingia
kaushikdehingia17@gmail.com
Department of Mathematics, Sonari College
Sonari 785690, Assam, India

Anusmita Das
anusmitadas87@gmail.com
Department of Mathematics, Gauhati University
Guwahati 781014, Assam, India

Soheil Salahshour
soheil.salahshour@eng.bau.edu.tr
Faculty of Engineering and Natural Sciences, Bahcesehir University
Istanbul 34349, Turkey

1. Introduction

The search for different wave structures of the $(2+1)$-dimensional chiral Schrödinger equation [1–7]

$$iu_t + c_1\left(u_{xx} + u_{yy}\right) + i\left(c_2\left(uu_x^* - u^*u_x\right) + c_3\left(uu_y^* - u^*u_y\right)\right)u = 0 \tag{1.1}$$

has achieved much attention in recent years and has been the topic of many studies. The origin of the 2D-CS equation goes back to its 1-dimensional version that was proposed by Nishino et al. [8] as a reduction of a model describing the edge states of the fractional quantum Hall effect [9]. Many scholars have spent their efforts to study the 2D-CS equation. Hosseini and Mirzazadeh [5] applied the Jacobi method to derive solitons and other solutions of the 2D-CS equation. Osman et al. [6] found a group of exact solutions of the 2D-CS equation using the Fan sub-equation method. Rezazadeh et al. [7] employed the extended rational sine-cosine/sinh-cosh methods to obtain traveling wave solutions of the 2D-CS equation. Very recently, Sulaiman and his colleagues [10] considered the 2D-CS equation with variable coefficients and obtained its complex wave solutions through a series of test functions. It is worth mentioning that u_t is the evolution term, c_1 is the coefficient of dispersion terms, and c_2 and c_3 are the coefficients of nonlinear terms. Additionally, Eq. (1.1) cannot possess the Painlevé test [1] and such a property indicates the significance of investigating its exact solutions.

The present article investigates different wave structures of the 2D-CS equation using the modified Kudryashov method [11–14]. This method utilizes a special finite series solution in the form [15]

$$U(\epsilon) = a_0 + \sum_{i=1}^{N}\left(\frac{K(\epsilon)}{1+K^2(\epsilon)}\right)^{i-1}\left(a_i\frac{K(\epsilon)}{1+K^2(\epsilon)} + b_i\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)}\right), \quad a_N \text{ or } b_N \neq 0,$$

which is different from that considered in its conventional version [16–19], namely,

$$U(\epsilon) = a_0 + \sum_{i=1}^{N} a_i K^i(\epsilon), \quad a_N \neq 0.$$

Such a selection provides other exact solutions of nonlinear PDEs which cannot be derived by the Kudryashov method. To address recent applications of the modified Kudryashov method in finding exact solutions of nonlinear PDEs, Hosseini et al. [11] adopted this method to look for exact solutions of a nonlinear high-order Schrödinger equation. Hosseini et al. [12] also applied this method to seek exact solutions of a 2D nonlinear Schrödinger system. More articles can be found in [20–29].

This paper is organized as follows: Section 2 presents an outline of the modified Kudryashov method along with several useful remarks. In Section 3, the real and imaginary portions of the 2D-CS equation are derived and then different wave structures of the 2D-CS equation are retrieved using the modified Kudryashov method. Besides, the dynamics of soliton solutions is investigated by representing a series of 3D-plots. Finally, Section 4 gives a review of the results.

2. Modified Kudryashov method

This section aims to present an outline of the modified Kudryashov method to retrieve different wave structures of nonlinear ODEs. To this end, the following nonlinear ODE is considered:

$$O(U, U', U'', \ldots) = 0, \quad U = U(\epsilon), \tag{2.1}$$

where $' = \frac{d}{d\epsilon}$.

The KM method supposes that the solution of Eq. (2.1) can be represented as

$$U(\epsilon)=a_0+\sum_{i=1}^{N}\left(\frac{K(\epsilon)}{1+K^2(\epsilon)}\right)^{i-1}\left(a_i\frac{K(\epsilon)}{1+K^2(\epsilon)}+b_i\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)}\right),\quad a_N \text{ or } b_N\neq 0, \tag{2.2}$$

where a_0, a_i, $i=1, 2, \ldots, N$, and b_i, $i=1, 2, \ldots, N$ are unknowns, N is established by the balance approach, and $K(\epsilon)$ is a function of the form

$$K(\epsilon)=\frac{1}{(A-B)\text{sinh}(\epsilon)+(A+B)\text{cosh}(\epsilon)}, \tag{2.3}$$

which solves the following Jacobi equation:

$$(K'(\epsilon))^2=K^2(\epsilon)\left(1-4ABK^2(\epsilon)\right).$$

By plugging Eqs. (2.2) and (2.3) into Eq. (2.1) and reorganizing the terms, we will reach a nonlinear algebraic set whose solution leads to different wave structures of Eq. (2.1).

Remark 1. When the balance number is $N=1$ and $b_1=a_2$, Eq. (2.2) can be written as

$$U(\epsilon)=a_0+a_1\frac{K(\epsilon)}{1+K^2(\epsilon)}+a_2\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)},\quad a_1 \text{ or } a_2\neq 0,$$

and so

$$U(\epsilon)=\frac{(a_0-a_2)K^2(\epsilon)+a_1K(\epsilon)+a_0+a_2}{K^2(\epsilon)+1},\quad a_1 \text{ or } a_2\neq 0. \tag{2.4}$$

Remark 2. By considering Eq. (2.3), Eq. (2.4) can be written as

$$\begin{aligned}U(\epsilon)=&\Big(\left(2A^2a_0+2A^2a_2-2B^2a_0-2B^2a_2\right)\sinh(\epsilon)\cosh(\epsilon)+(Aa_1-Ba_1)\sinh(\epsilon)+\\&+\left(2A^2a_0+2A^2a_2+2B^2a_0+2B^2a_2\right)(\cosh(\epsilon))^2+(Aa_1+Ba_1)\cosh(\epsilon)-A^2a_0-A^2a_2+\\&+2ABa_0+2ABa_2-B^2a_0-B^2a_2+a_0-a_2\Big)\Big/\Big(\left(2A^2-2B^2\right)\cosh(\epsilon)\sinh(\epsilon)+\\&+\left(2A^2+2B^2\right)(\cosh(\epsilon))^2-A^2+2AB-B^2+1\Big),\quad a_1 \text{ or } a_2\neq 0.\end{aligned} \tag{2.5}$$

Remark 3. By considering

$$A=-\frac{1}{2B},\quad a_0=\mp b,\quad a_1=0,\quad a_2=\pm b,$$

from Eq. (2.5), one has

$$U_{1,2}(\epsilon)=\pm\frac{8bB^2}{(8B^4-2)\sinh(\epsilon)\cosh(\epsilon)+(-8B^4-2)(\cosh(\epsilon))^2+4B^4+1}.$$

Remark 4. By considering

$$B=0,\quad a_0=0,\quad a_1=\pm b,\quad a_2=0,$$

from Eq. (2.5), one has

$$U_{3,4}(\epsilon)=\pm\frac{bA(\sinh(\epsilon)+\cosh(\epsilon))}{2A^2\sinh(\epsilon)\cosh(\epsilon)+2A^2(\cosh(\epsilon))^2-A^2+1}.$$

Remark 5. By considering

$$B = 0, \quad a_0 = 0, \quad a_1 = 0, \quad a_2 = \pm b,$$

from Eq. (2.5), one has

$$U_{5,6}(\epsilon) = \pm \frac{b\left(A^4 + 4A^2 \sinh(\epsilon)\cosh(\epsilon) - 1\right)}{A^4 + 4A^2(\cosh(\epsilon))^2 - 2A^2 + 1}.$$

Remark 6. By considering

$$B = 0, \quad a_0 = 0, \quad a_1 = \pm b, \quad a_2 = c,$$

from Eq. (2.5), one has

$$U_{7,8}(\epsilon) = \frac{\left(A^2 c + c\right)\sinh(\epsilon) + \left(A^2 c - c\right)\cosh(\epsilon) \pm bA}{\left(A^2 - 1\right)\sinh(\epsilon) + \left(A^2 + 1\right)\cosh(\epsilon)}.$$

3. 2D-CS equation and its different wave structures

This section aims to derive different wave structures of the 2D-CS equation through the modified Kudryashov method. For this purpose, the following traveling wave transformation is used:

$$u(x,\, y,\, t) = U(\epsilon)e^{i(\kappa_2 x + \lambda_2 y + \mu_2 t)}, \quad \epsilon = \kappa_1 x + \lambda_1 y - \mu_1 t, \tag{3.1}$$

where κ_2 and λ_2 are frequencies in the x- and y-directions, μ_2 is the wave frequency, μ_1 is the wave velocity, and the other parameters are real constants. The traveling wave transformation (3.1) reduces the 2D-CS equation to

$$c_1\left(\kappa_1^2 + \lambda_1^2\right)\frac{d^2U(\epsilon)}{d\epsilon^2} - \left(c_1\kappa_2^2 + c_1\lambda_2^2 + \mu_2\right)U(\epsilon) + 2\left(c_2\kappa_2 + c_3\lambda_2\right)U^3(\epsilon) = 0, \tag{3.2}$$

where the wave velocity is $\mu_1 = 2c_1(\kappa_1\kappa_2 + \lambda_1\lambda_2)$.

Owing to the terms $\frac{d^2U(\epsilon)}{d\epsilon^2}$ and $U^3(\epsilon)$, we find the balance number as $N = 1$. Accordingly, the nontrivial solution of Eq. (3.2) can be expressed as

$$U(\epsilon) = a_0 + a_1\frac{K(\epsilon)}{1 + K^2(\epsilon)} + a_2\frac{1 - K^2(\epsilon)}{1 + K^2(\epsilon)}, \quad a_1 \text{ or } a_2 \neq 0, \tag{3.3}$$

where a_0, a_1, and a_2 are unknowns. By plugging Eq. (3.3) into Eq. (3.2) and reorganizing the terms, we will reach a nonlinear algebraic set in the form

$$\begin{gathered}
-2a_0^3c_2\kappa_2 - 2a_0^3c_3\lambda_2 - 6a_0^2a_2c_2\kappa_2 - 6a_0^2a_2c_3\lambda_2 - 6a_0a_2^2c_2\kappa_2 - 6a_0a_2^2c_3\lambda_2 - 2a_2^3c_2\kappa_2 - \\
-2a_2^3c_3\lambda_2 + a_0c_1\kappa_2^2 + a_0c_1\lambda_2^2 + a_2c_1\kappa_2^2 + a_2c_1\lambda_2^2 + a_0\mu_2 + a_2\mu_2 = 0, \\
-6a_0^2a_1c_2\kappa_2 - 6a_0^2a_1c_3\lambda_2 - 12a_0a_1a_2c_2\kappa_2 - 12a_0a_1a_2c_3\lambda_2 - 6a_1a_2^2c_2\kappa_2 - \\
-6a_1a_2^2c_3\lambda_2 - a_1c_1\kappa_1^2 + a_1c_1\kappa_2^2 - a_1c_1\lambda_1^2 + a_1c_1\lambda_2^2 + a_1\mu_2 = 0, \\
-6a_0^3c_2\kappa_2 - 6a_0^3c_3\lambda_2 - 6a_0^2a_2c_2\kappa_2 - 6a_0^2a_2c_3\lambda_2 - 6a_0a_1^2c_2\kappa_2 - 6a_0a_1^2c_3\lambda_2 + 6a_0a_2^2c_2\kappa_2 + \\
+6a_0a_2^2c_3\lambda_2 - 6a_1^2a_2c_2\kappa_2 - 6a_1^2a_2c_3\lambda_2 + 6a_2^3c_2\kappa_2 + 6a_2^3c_3\lambda_2 + 3a_0c_1\kappa_2^2 + 3a_0c_1\lambda_2^2 + \\
+8a_2c_1\kappa_1^2 + a_2c_1\kappa_2^2 + 8a_2c_1\lambda_1^2 + a_2c_1\lambda_2^2 + 3a_0\mu_2 + a_2\mu_2 = 0,
\end{gathered}$$

$$8ABa_1c_1\kappa_1^2+8ABa_1c_1\lambda_1^2-12a_0^2a_1c_2\kappa_2-12a_0^2a_1c_3\lambda_2-2a_1^3c_2\kappa_2-2a_1^3c_3\lambda_2+12a_1a_2^2c_2\kappa_2+ \\ +12a_1a_2^2c_3\lambda_2+6a_1c_1\kappa_1^2+2a_1c_1\kappa_2^2+6a_1c_1\lambda_1^2+2a_1c_1\lambda_2^2+2a_1\mu_2=0,$$
$$-48ABa_2c_1\kappa_1^2-48ABa_2c_1\lambda_1^2-6a_0^3c_2\kappa_2-6a_0^3c_3\lambda_2+6a_0^2a_2c_2\kappa_2+6a_0^2a_2c_3\lambda_2-6a_0a_1^2c_2\kappa_2- \\ -6a_0a_1^2c_3\lambda_2+6a_0a_2^2c_2\kappa_2+6a_0a_2^2c_3\lambda_2+6a_1^2a_2c_2\kappa_2+6a_1^2a_2c_3\lambda_2-6a_2^3c_2\kappa_2-6a_2^3c_3\lambda_2+ \\ +3a_0c_1\kappa_2^2+3a_0c_1\lambda_2^2-8a_2c_1\kappa_1^2-a_2c_1\kappa_2^2-8a_2c_1\lambda_1^2-a_2c_1\lambda_2^2+3a_0\mu_2-a_2\mu_2=0,$$
$$-24ABa_1c_1\kappa_1^2-24ABa_1c_1\lambda_1^2-6a_0^2a_1c_2\kappa_2-6a_0^2a_1c_3\lambda_2+12a_0a_1a_2c_2\kappa_2+12a_0a_1a_2c_3\lambda_2- \\ -6a_1a_2^2c_2\kappa_2-6a_1a_2^2c_3\lambda_2-a_1c_1\kappa_1^2+a_1c_1\kappa_2^2-a_1c_1\lambda_1^2+a_1c_1\lambda_2^2+a_1\mu_2=0,$$
$$16ABa_2c_1\kappa_1^2+16ABa_2c_1\lambda_1^2-2a_0^3c_2\kappa_2-2a_0^3c_3\lambda_2+6a_0^2a_2c_2\kappa_2+6a_0^2a_2c_3\lambda_2-6a_0a_2^2c_2\kappa_2- \\ -6a_0a_2^2c_3\lambda_2+2a_2^3c_2\kappa_2+2a_2^3c_3\lambda_2+a_0c_1\kappa_2^2+a_0c_1\lambda_2^2-a_2c_1\kappa_2^2-a_2c_1\lambda_2^2+a_0\mu_2-a_2\mu_2=0.$$

Through employing the Maple software, from the above system, the following cases are generated:

Case 1.

$$A=-\frac{1}{2B},\quad a_0=\mp\underbrace{\sqrt{-\frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}}_{b},\quad a_1=0,\quad a_2=\pm\underbrace{\sqrt{-\frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}}_{b},$$
$$\mu_2=4c_1\kappa_1^2-c_1\kappa_2^2+4c_1\lambda_1^2-c_1\lambda_2^2.$$

Thus, the following soliton solutions to the 2D-CS equation are derived:

$$u_{1,2}(x,\,y,\,t)= \\ =\pm 8\sqrt{-\frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}B^2\bigg/\Big(\big(8B^4-2\big)\sinh(\kappa_1x+\lambda_1y-\mu_1t)\cosh(\kappa_1x+\lambda_1y-\mu_1t)+ \\ +\big(-8B^4-2\big)\left(\cosh(\kappa_1x+\lambda_1y-\mu_1t)\right)^2+4B^4+1\Big)\,e^{i(\kappa_2x+\lambda_2y+\mu_2t)},$$

where

$$\mu_1=2c_1(\kappa_1\kappa_2+\lambda_1\lambda_2),\quad \mu_2=4c_1\kappa_1^2-c_1\kappa_2^2+4c_1\lambda_1^2-c_1\lambda_2^2,\quad \frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}<0.$$

Case 2.

$$B=0,\quad a_0=0,\quad a_1=\pm 2\underbrace{\sqrt{-\frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}}_{b},\quad a_2=0,\quad \mu_2=c_1\kappa_1^2-c_1\kappa_2^2+c_1\lambda_1^2-c_1\lambda_2^2.$$

Therefore, the following soliton solutions to the 2D-CS equation are obtained:

$$u_{3,4}(x,\,y,\,t)=\pm 2\sqrt{-\frac{-c_1\kappa_1^2-c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}A(\sinh(\kappa_1x+\lambda_1y-\mu_1t)+ \\ +\cosh(\kappa_1x+\lambda_1y-\mu_1t))\Big/\left(2A^2\sinh(\kappa_1x+\lambda_1y-\mu_1t)\cosh(\kappa_1x+\lambda_1y-\mu_1t)+\right. \\ \left.+2A^2(\cosh(\kappa_1x+\lambda_1y-\mu_1t))^2-A^2+1\right)e^{i(\kappa_2x+\lambda_2y+\mu_2t)},$$

where

$$\mu_1 = 2c_1(\kappa_1\kappa_2 + \lambda_1\lambda_2), \quad \mu_2 = c_1\kappa_1^2 - c_1\kappa_2^2 + c_1\lambda_1^2 - c_1\lambda_2^2, \quad \frac{-c_1\kappa_1^2 - c_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2} < 0.$$

Case 3.

$$B = 0, \quad a_0 = 0, \quad a_1 = 0, \quad a_2 = \pm\underbrace{\sqrt{-\frac{c_1\kappa_1^2 + c_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2}}}_{b}, \quad \mu_2 = -c_1\left(2\kappa_1^2 + \kappa_2^2 + 2\lambda_1^2 + \lambda_2^2\right).$$

Consequently, the following soliton solutions to the 2D-CS equation are obtained:

$$u_{5,6}(x,\, y,\, t) = \pm\frac{\sqrt{-\frac{c_1\kappa_1^2+c_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}\left(A^4 + 4A^2\sinh(\kappa_1 x + \lambda_1 y - \mu_1 t)\cosh(\kappa_1 x + \lambda_1 y - \mu_1 t) - 1\right)}{A^4 + 4A^2(\cosh(\kappa_1 x + \lambda_1 y - \mu_1 t))^2 - 2A^2 + 1}\times$$
$$\times\, e^{i(\kappa_2 x + \lambda_2 y + \mu_2 t)},$$

where

$$\mu_1 = 2c_1(\kappa_1\kappa_2 + \lambda_1\lambda_2), \quad \mu_2 = -c_1\left(2\kappa_1^2 + \kappa_2^2 + 2\lambda_1^2 + \lambda_2^2\right), \quad \frac{c_1\kappa_1^2 + c_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2} < 0.$$

Case 4.

$$B = 0, \quad a_0 = 0, \quad a_1 = \pm\underbrace{\sqrt{-\frac{-c_1\kappa_1^2 - c_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2}}}_{b}, \quad a_2 = \underbrace{\sqrt{-\frac{c_1\kappa_1^2 + c_1\lambda_1^2}{4c_2\kappa_2 + 4c_3\lambda_2}}}_{c},$$
$$\mu_2 = -\frac{1}{2}\left(\kappa_1^2 + 2\kappa_2^2 + \lambda_1^2 + 2\lambda_2^2\right)c_1.$$

Accordingly, the following exact solutions to the 2D-CS equation are achieved:

$$u_{7,8}(x,\, y,\, t) = \frac{\left(A^2c + c\right)\sinh(\kappa_1 x + \lambda_1 y - \mu_1 t) + \left(A^2c - c\right)\cosh(\kappa_1 x + \lambda_1 y - \mu_1 t) \pm bA}{(A^2 - 1)\sinh(\kappa_1 x + \lambda_1 y - \mu_1 t) + (A^2 + 1)\cosh(\kappa_1 x + \lambda_1 y - \mu_1 t)}\times$$
$$\times\, e^{i(\kappa_2 x + \lambda_2 y + \mu_2 t)},$$

where

$$b = \sqrt{-\frac{-c_1\kappa_1^2 - c_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2}},$$
$$c = \sqrt{-\frac{c_1\kappa_1^2 + c_1\lambda_1^2}{4c_2\kappa_2 + 4c_3\lambda_2}},$$
$$\mu_1 = 2c_1(\kappa_1\kappa_2 + \lambda_1\lambda_2),$$
$$\mu_2 = -\frac{1}{2}\left(\kappa_1^2 + 2\kappa_2^2 + \lambda_1^2 + 2\lambda_2^2\right)c_1.$$

REMARK. It should be noted that, by applying the Kudryashov method, one arrives at a nonlinear algebraic set in the form

$$\begin{gathered}2a_0^3c_2\kappa_2 + 2a_0^3c_3\lambda_2 - a_0c_1\kappa_2^2 - a_0c_1\lambda_2^2 - a_0\mu_2 = 0,\\ 6a_0^2a_1c_2\kappa_2 + 6a_0^2a_1c_3\lambda_2 + a_1c_1\kappa_1^2 - a_1c_1\kappa_2^2 + a_1c_1\lambda_1^2 - a_1c_1\lambda_2^2 - a_1\mu_2 = 0,\\ 6a_0a_1^2c_2\kappa_2 + 6a_0a_1^2c_3\lambda_2 = 0,\\ -8ABa_1c_1\kappa_1^2 - 8ABa_1c_1\lambda_1^2 + 2a_1^3c_2\kappa_2 + 2a_1^3c_3\lambda_2 = 0,\end{gathered}$$

where its solution gives

$$a_0 = 0, \quad a_1 = \pm\sqrt{-\frac{-4ABc_1\kappa_1^2 - 4ABc_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2}}, \quad \mu_2 = c_1\kappa_1^2 - c_1\kappa_2^2 + c_1\lambda_1^2 - c_1\lambda_2^2.$$

Subsequently, the following soliton solutions to the 2D-CS equation are obtained:

$$u_{1,2}(x,\, y,\, t) = \pm\frac{\sqrt{-\frac{-4ABc_1\kappa_1^2-4ABc_1\lambda_1^2}{c_2\kappa_2+c_3\lambda_2}}}{(A-B)\sinh(\kappa_1x + \lambda_1y - \mu_1t) + (A+B)\cosh(\kappa_1x + \lambda_1y - \mu_1t)}e^{i(\kappa_2x+\lambda_2y+\mu_2t)},$$

where

$$\begin{gathered}\mu_1 = 2c_1(\kappa_1\kappa_2 + \lambda_1\lambda_2),\\ \mu_2 = c_1\kappa_1^2 - c_1\kappa_2^2 + c_1\lambda_1^2 - c_1\lambda_2^2,\\ \frac{-4ABc_1\kappa_1^2 - 4ABc_1\lambda_1^2}{c_2\kappa_2 + c_3\lambda_2} < 0.\end{gathered}$$

Now, the authors are interested in analyzing the dynamics of soliton solutions derived above by representing a series of 3D-plots. To this end, the first soliton derived using the MK method has been plotted in Figure 1 for $B = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = 0.5$, $\kappa_2 = -0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$. Figure 2 represents the fifth soliton obtained through the MK method for $A = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = -0.25$, $\kappa_2 = -0.25$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$. The first soliton derived using the Kudryashov method has been portrayed in Figure 3 for $A = 2$, $B = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = 0.5$, $\kappa_2 = -0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$. It should be stated that the first, second, and third figures signify the bright, dark, and bright solitons, respectively. Furthermore, the MK method is capable of retrieving both bright and dark solitons.

As a specific feature, the bright and dark solitons derived using the MK method move in opposite directions. To show such a feature, the following figures are considered.

4. Conclusion

In the present paper, the $(2+1)$-dimensional chiral Schrödinger equation with applications in mathematical physics was considered and explored successfully. First, a traveling wave transformation was adopted to derive the real and imaginary portions of the 2D-CS equation, then the second-order nonlinear ODE in the real domain was solved through the modified Kudryashov method and the symbolic computation package. As an achievement, several bright and dark solitons to the 2D-CS equation were formally extracted. In the end, the dynamics of soliton solutions was examined by establishing a series of 3D-plots.

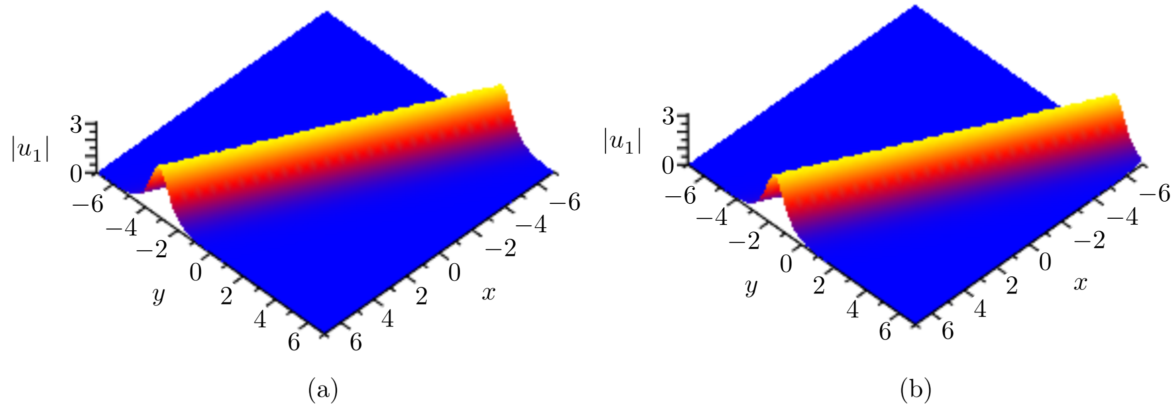


Fig. 1. The first soliton derived using the MK method for $B = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = 0.5$, $\kappa_2 = -0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$

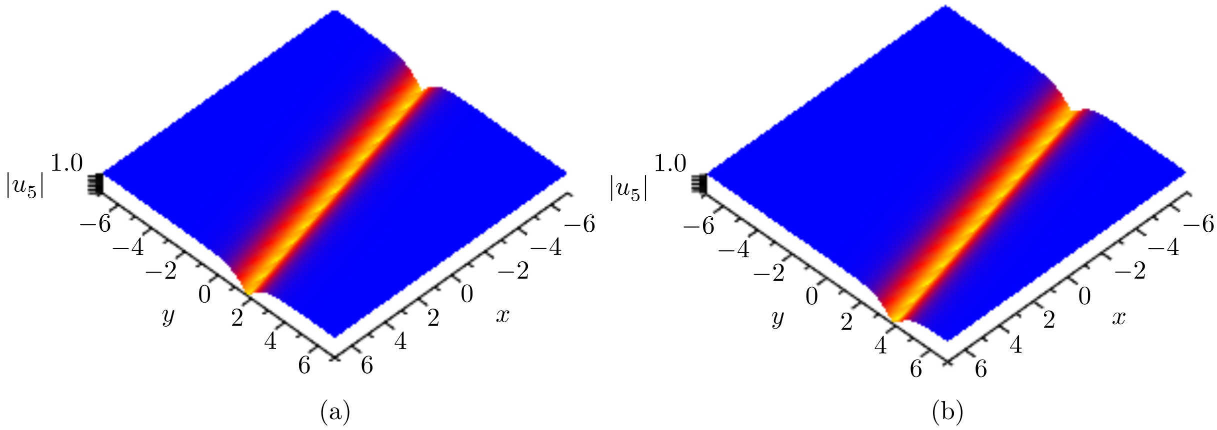


Fig. 2. The fifth soliton obtained through the MK method for $A = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = -0.25$, $\kappa_2 = -0.25$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$

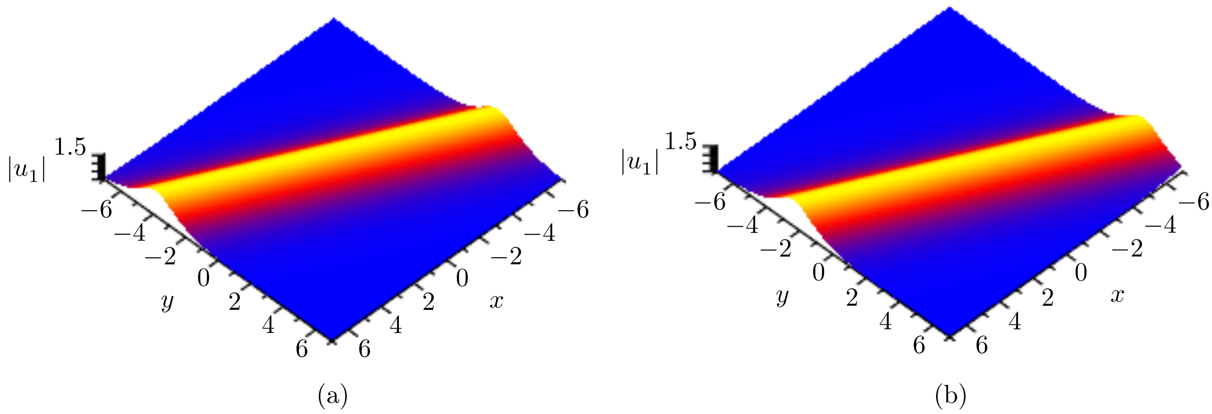
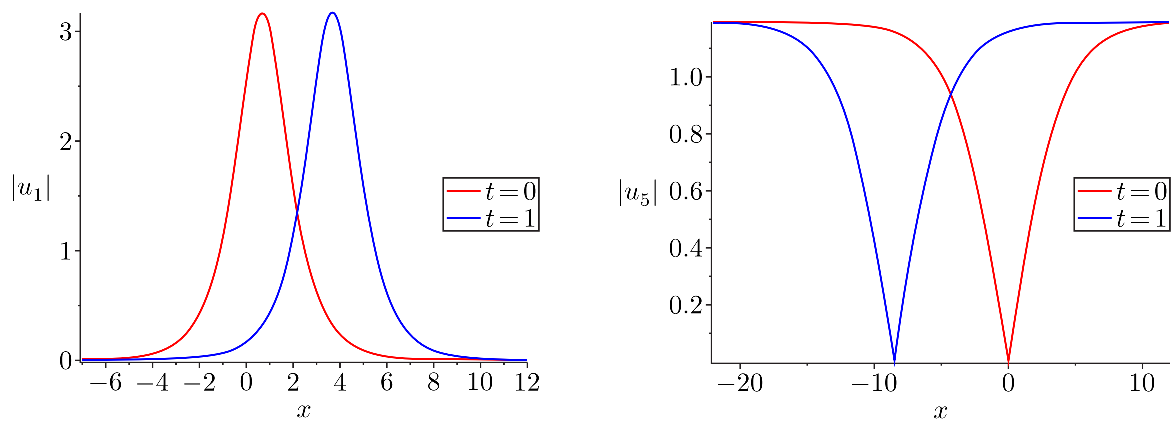


Fig. 3. The first soliton derived using the Kudryashov method for $A = 2$, $B = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = 0.5$, $\kappa_2 = -0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$, (a) $t = 0$ and (b) $t = 1$



(a) (b)

Fig. 4. (a) The first soliton derived using the MK method for $B = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = 0.5$, $\kappa_2 = -0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$, and $y = 0$; (b) The fifth soliton obtained through the MK method for $A = 1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $\kappa_1 = -0.25$, $\kappa_2 = -0.25$, $\lambda_1 = 1$, $\lambda_2 = 1$, and $y = 0$

Data availability

The authors declare that this research is purely theoretical and is not associated with any data.

Conflict of interest

The authors declare no conflict of interest.

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